A low-energy compatible SU(4)-type model for vector leptoquarks of mass $\leq 1\,\mathrm{TeV}$

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Abstract. The Standard Model is extended by a $SU(2)_L$ singlet of vector leptoquarks. An additional $SU(4)$ gauge symmetry between right-handed up quarks and right-handed leptons is introduced to render the model renormalizable. The arrangement is made in such a way that no conflict with low energy restrictions is encountered. The $SU(2)_L$ singlet mediates interactions between the right-handed leptons and up type quarks for which only moderate low energy restrictions $M_{LQ}/g_{LQ} >$ few hundred GeV exist. However, it is *not* a candidate to explain the anomalous HERA data at large Q^2 because theoretical reasons imply that $g_{LQ} \geq g_s$ which would give a much stronger anomalous HERA effect. We furthermore argue that the inequality $g_{LQ} \geq g_s$ is a general feature of consistent vector leptoquark models. Although our model is not relevant for HERA, it is interesting per se as a description of leptoquarks of mass $\leq 1\,\mathrm{TeV}$ consistent with all low-energy requirements.

1 Introduction

There has been an increasing interest in leptoquarks of mass $M_{LQ} \sim$ few hundred GeV in the last months, due to the exciting possibility of observing such particles at HERA [1–4]. Single leptoquarks may be produced in electron proton collisions directly in an s-channel process $eq \to LQ$ whereas in proton–proton collisions they contribute more indirectly via t-channel exchange or are pair-produced. Correspondingly, leptoquarks seen at HERA need not necessarily satisfy the Tevatron bounds [5], because the processes and couplings involved are different.

On the theoretical side there are many extensions of the standard model which predict the existence of leptoquarks with masses which could be of the order of a few hundred GeV [6,7]. However, phenomenological considerations [8–10] show that most of these models are in conflict with low energy data. Either they induce proton decay or various FCNC processes or they enhance leptonic decays of pseudoscalar mesons. The phenomenological restrictions can usually be expressed in terms of the ratio M_{LQ}/g_{LQ} where g_{LQ} is the coupling of the leptoquarks to quarks and leptons.

2 Low energy constraints

More in detail, the processes that lead to the strongest bounds on the leptoquark mass and couplings are

(i) *proton decay*
This is induced when the leptoquark has diquark cou-

plings as well, so that processes $qq \to ql$ etc are possible. Leptoquarks of this type are out of reach of any future collider.

(ii) *flavor changing neutral current processes*
These are induced when the leptoquark couples to more than one generation in the lepton and/or the quark sector. The strongest bound arises from the decay $K_L \to \mu e$ which is induced by exchange of a leptoquark in the t-channel and typically given by $M_{LQ}/g_{LQ} \geq 100\,\mathrm{TeV}$ [8].

(iii) *leptonic decays of pions and other pseudoscalars*
This bound is particularly strong for leptoquarks that couple both to left-handed and right-handed quarks, namely $M_{LQ}/g_{LQ} \geq 100\,\mathrm{TeV}$ [8].

(iv) *other processes*
There are a number of other processes like D-decays, $K^0\bar{K}^0$ mixing, μ-decays, τ-decays [8] and atomic parity violation [10,11], all of which give weaker constraints to leptoquark masses and couplings and are compatible with a leptoquark $M_{LQ}/g_{LQ} \sim O(1)\,\mathrm{TeV}$. Thus, these processes are not in contradiction with a low lying leptoquark of mass ~ 200 GeV provided the leptoquark coupling is sufficiently weak and not of the order of the strong coupling constant. The most interesting among these restrictions are perhaps the atomic experiments, because leptoquarks give parity violating contributions like $\frac{g_{LQ}^2}{M_{LQ}^2}(\bar{e}\gamma_\mu\gamma_5 e)(\bar{q}\gamma_\mu q)$ to the interactions of electrons and quarks in ordinary atoms [10].

In the model presented below the bounds (i-iii) are avoided. There are no proton decays and the FCNC processes involving Kaons are avoided by the leptoquarks coupling to up- but not to down-type quarks. The restrictions from D-decays are much less severe than from K-decays, typically given by $M_{LQ}/g_{LQ} \geq O(1)\,$TeV [8]. Furthermore, the strong bound from pion decays is avoided because the leptoquarks couple chirally, and in particular they couple only to right-handed quarks. Note also that there is no "CKM-type" mixing in the model.

3 General analysis

Possible leptoquarks interactions have been analyzed in [12] in a model independent way from a purely phenomenological point of view. Because of their coupling to quarks, all leptoquarks carry color (in the fundamental representation). Furthermore, all leptoquark fields have dimension 1 and integer spin (0 or 1), i.e. they are either scalar or vector fields. Depending on whether they interact with a fermion–antifermion or a fermion–fermion system they carry fermion number $F = 3B + L = 0$ or -2. Using the assumption that the leptoquark interactions respect the symmetries of the standard model the most general effective Lagrangian involving leptoquarks was derived in [12]. These relatively mild assumptions lead to a variety of leptoquarks. In the model presented below just one of these is selected by requiring the following principles

– *gauge principle*
  this assumes that the leptoquarks themselves arise as vector bosons of a new gauge group. This excludes all scalar leptoquarks from the list. Indeed, leptoquarks are naturally gauge bosons and as such they appear in many extensions of the standard model. Scalar leptoquarks appear in conjunction with the corresponding Higgs mechanism (see below) or as superpartners in supersymmetric theories [13,?,3,4]. It is true that Tevatron data seem to exclude vector leptoquarks below 300 GeV [5]. Therefore, in this article we explore the theoretical possibility of a vector leptoquark of mass 300 GeV $\leq m_{LQ} \leq 1\,$TeV.
– *universality*
  this implies that the leptoquark couplings to all families are the same. It is a reasonable assumption in view of the known universality of all the other gauge interactions.
– *vanishing fermion number*
  this leaves only the $F = 0$ leptoquarks in the list. In the appendix there will be a short discussion about what happens if this assumption is given up. It turns out, that one can construct a consistent $F = -2$ vector leptoquark model based on the gauge group $SU(5) \times SU(3) \times SU(2) \times U(1)$, cf. the appendix.

Together with the low energy constraints, these principles are so strong that only one of the leptoquarks passes the requirements, namely an $F = 0$ vector particle V_μ^i with quantum numbers $(3, 1, \frac{5}{3})$ under $SU(3)_c \times SU(2)_L \times$

$U(1)_Y$ and lepton–quark interactions $g_{LQ}\bar{u}_R^i\gamma^\mu e_R V_\mu^i$ +c.c. (i=color index). A goal of this letter is to embed this particle and its interaction into a renormalizable (and thus consistent) extension of the standard model. A Higgs mechanism will be invoked to obtain the leptoquark mass. The minimal extension of the standard model which includes the fields V_μ^i is by an $SU(4)$ gauge group which acts on the right-handed quartet p_R formed by e_R and u_R^i, i=1,2,3. V_μ^i are thus leptoquarks of the Pati–Salam type, but without interactions to d-type quarks and to left-handed fermions. The total symmetry group of the model is $SU(4) \times SU(3) \times SU(2)_L \times U(1)_X$. Let us write down the Lagrangian:

$$L = \bar{p}_R i\gamma^\mu[\partial_\mu + ig_1 X(p_R)C_\mu + ig_4 R_\mu^a \frac{\rho_a}{2}]p_R \qquad (1)$$

$$+\bar{d}_R i\gamma^\mu[\partial_\mu + ig_1 Q(d_R)C_\mu + ig_3 L_\mu^a \frac{\lambda_a}{2}]d_R$$

$$+\bar{l}_L i\gamma^\mu[\partial_\mu + ig_1 Y(l_L)C_\mu + ig_2 W_\mu^a \frac{\tau_a}{2}]l_L$$

$$+\bar{q}_L i\gamma^\mu[\partial_\mu + ig_1 Y(q_L)C_\mu + ig_2 W_\mu^a \frac{\tau_a}{2} + ig_3 L_\mu^a \frac{\lambda_a}{2}]q_L$$

$$-\frac{1}{4}R_{\mu\nu}^a R^{\mu\nu a} - \frac{1}{4}L_{\mu\nu}^a L^{\mu\nu a} - \frac{1}{4}W_{\mu\nu}^a W^{\mu\nu a} - \frac{1}{4}C_{\mu\nu}C^{\mu\nu}$$

Here q_L and l_L are the left-handed quark and lepton doublets, and e_R, d_R and u_R the right-handed charged leptons, down- and up-type quarks, respectively. Color, weak isospin and generation indices have been suppressed. $X(p_R)$ is the U(1) charge of the quartet $p_R = (u_R, e_R)$. It will be fixed later to be $X(p_R) = \frac{1}{4}$ by requiring that the electromagnetic coupling comes out right. For all other fermion fields, l_L, d_R and q_L, the X-charge agrees with the weak hypercharge. C_μ is the $U(1)_X$ gauge field which will mix with the other neutral fields of the model, W_μ^3 and R_μ^{15}. R_μ, L_μ and W_μ are the gauge bosons of the $SU(4)$, $SU(3)$ and $SU(2)$, respectively, with gauge couplings g_4, g_3 and g_2. The algebra of $SU(4)$ is spanned by the matrices ρ_a, a=1,...,15, where ρ_1,...,ρ_8 are the SU(3) λ-matrices. For example, ρ_{15} is given by $\frac{\rho_{15}}{2} = \frac{1}{\sqrt{24}}\text{diag}(-1,-1,-1,3)$. One can write

$$R_\mu = R_\mu^a \frac{\rho_a}{2} = \frac{1}{\sqrt{2}}\begin{pmatrix} \hat{R}_\mu - \frac{S_\mu \times 1}{\sqrt{12}} & V_\mu \\ V_\mu^+ & \sqrt{\frac{3}{4}}S_\mu \end{pmatrix}, \qquad (2)$$

where group indices have been suppressed. One sees that besides the leptoquarks V_μ, there is an octet $\hat{R}_\mu$ and a singlet $S_\mu := R_\mu^{15}$ of vector bosons. $\hat{R}_\mu$ will mix with the SU(3) octet L_μ to form 8 massless gluons and 8 massive 'axigluon' states.

The quantum number assignments for the fermions can be found in Table 1. A family symmetry is assumed which is only broken by fermion mass terms. As can be seen from Table 1, there are big differences as compared to the Pati–Salam model. The main difference is that not all possible leptons and quarks are put into a $SU(4)$ multiplet, but only the right-handed electron and up–quark. Note that this is a *chiral* model, i.e. left- and right-handed fermions

Table 1. Quantum number assignments

	$SU(4)$	$SU(3)$	$SU(2)_L$	$U(1)_X$
q_L	1	3	2	$\frac{1}{6}$
l_L	1	1	2	$-\frac{1}{2}$
p_R	4	1	1	$\frac{1}{4}$
d_R	1	3	1	$-\frac{1}{3}$

behave non-symmetric, so as to embed the weak interactions in the model.

Higgs–terms have been omitted in the Lagrangian. They will be discussed later, and as usual, they will provide boson and fermion masses. Essentially, there will be one Higgsfield, $H(\overline{4}, 3, 1)$ with vev v, which will break $SU(3) \times SU(4)$ to color $SU(3)$, and three other Higgs fields, which break $SU(2)_L$ and give masses to the fermions. The leptoquark masses are then of the order $g_4 v$ whereas the W and Z mass turn out to be as in the Standard Model.

4 Color sector

Those Higgs interactions should break the $SU(3) \times SU(4)$ symmetry down to the diagonal $SU(3)_c$ in a similar fashion than happens in the so–called chiral–color models [14] which are based on $SU(3)_L \times SU(3)_R$. In those models, the gauge fields $\hat{R}_\mu$ and L_μ couple to right- and left-handed currents and are rotated in order to get the QCD couplings to the gluons G_μ right,

$$L_\mu = c_\theta N_\mu + s_\theta G_\mu$$
$$\hat{R}_\mu = -s_\theta N_\mu + c_\theta G_\mu \qquad (3)$$

The 'right-handed' bosons are written with a hat here in order to make the analogy with our model clear. N_μ are the 'axigluons' which have to become heavy by a suitable Higgs mechanism which breaks $SU(3)_L \times SU(3)_R$ to the diagonal $SU(3)$. In the limit that the $SU(3)_L$ and the $SU(3)_R$ gauge couplings are identical, the N_μ couple purely axially to fermions. Hence the name axigluons. Present Tevatron restrictions on axigluons are such that an axigluon with mass 500 GeV would be compatible with almost all bounds [15,16].

In the SU(4) case at hand one can proceed analogously. The relevant interactions of the quarks are given by

$$L_{strong} = g_3 \bar{d}_R \gamma_\mu L^\mu d_R + g_3 \bar{d}_L \gamma_\mu L^\mu d_L$$
$$+ g_3 \bar{u}_L \gamma_\mu L^\mu u_L + g_4 \bar{u}_R \gamma_\mu \hat{R}^\mu u_R. \qquad (4)$$

Inserting (3), one can prove that the ordinary gluon interactions are reproduced if

$$g_s = g_3 s_\theta = g_4 c_\theta \qquad (5)$$

or, equivalently,

$$g_s^{-2} = g_3^{-2} + g_4^{-2}. \qquad (6)$$

An immediate consequence of these relations is that both g_3 and g_4 are necessarily larger than the QCD coupling, i.e. for scales below 1 TeV one has $g_{3,4} \gtrsim O(1)$. This is undesired in view of the anomalous DESY–HERA data because the leptoquark coupling to fermions is $g_{LQ} = g_4$ and the HERA data require a smaller coupling. Furthermore, one has low-energy constraints $M_{LQ}/g_{LQ} \gtrsim O(1)\,\mathrm{TeV}$ which imply that the leptoquark mass M_V in our model is closer to 1 TeV than to 200 GeV. It should be stressed that the relation $g_{LQ} \gtrsim O(1)$ is a characteristic feature of the class of models discussed in this letter.

5 Higgs sector

In principle, there is some ambiguity in choosing the Higgs multiplets and the Higgs potential. In the following we present one possibility which is consistent with the Standard Model at low energies. The Higgs fields in this scenario have a rather complicated structure. From the quantum number assignments in Table 1 it is evident that one needs different Higgs multiplets for the various fermions to become heavy. There is a Higgs field $H_e(\overline{4}, 1, 2, -\frac{1}{2} - X(p_R))$ with vev $v_e \delta_{\bar{\alpha}4} \delta_{i2}$ for the lepton masses, a Higgs field $H_u(\overline{4}, 3, 2, \frac{1}{6} - X(p_R))$ with vev $v_u \delta_{\bar{\alpha}\beta} \delta_{i1}$ for the up quark masses and a Higgs field $H_d(1, 1, 2, \frac{1}{2})$ with vev $v_d \delta_{i2}$ for the down quark masses. Here, $\bar{\alpha}$, β and i are $SU(4)$, $SU(3)$ and $SU(2)_L$ indices, respectively. Altogether, the Yukawa terms of the Lagrangian are given by

$$L_{Yuk} = h_e H_e \bar{l}_L p_R + h_u H_u \bar{q}_L p_R + h_d H_d \bar{q}_L d_R + c.c. \qquad (7)$$

It is interesting to note that apart from giving masses to the fermions, part of these interactions have the form of interactions of scalar leptoquarks. However, due to the smallness of the Yukawa couplings, they are relevant (if at all) only for the third family. Of course, (7) induces other interactions between fermions and Higgs fields as well. If the Higgs masses are not too large, there may be some relevance for interactions of the top quark, because the top quark Yukawa coupling is not small. For example, in (7) there are colored Higgs fields which mediate interactions between bottom- and top-quark.

Note further, that H_d has the quantum numbers of the Standard Model Higgs field. A reasonable outcome for the vacuum expectation values of the three Higgs fields would certainly be $v_u : v_d : v_e \sim m_t : m_b : m_\tau$. However, it will be shown that this is not compatible with the requirement that γ and Z couplings to fermions are as given in the Standard Model. In fact, to achieve this goal, we will be forced to introduced yet another Higgs multiplet, $H(\overline{4}, 3, 1, \frac{5}{12})$ with vev $v \delta_{\bar{\alpha}\beta}$ which breaks $SU(4) \times SU(3)$ to $SU(3)_c$ [1]. Although H_u in principle does the same job, it is incompatible with the correct Z couplings to fermions (see below). A typical solution will be that v is of the order of the leptoquark mass, and v_d, v_u and v_e of the order of the electroweak symmetry breaking scale or somewhat

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[1] The X-charge $\frac{5}{12}$ of H is fixed by the requirement that the components $H_{\bar{\beta}\beta}$, $\bar{\beta} = 1, 2, 3$ are neutral

$$\frac{1}{2}M^2 = \begin{pmatrix} \frac{g_1^2}{4}\left(\frac{25}{12}v^2 + \frac{3}{36}v_u^2 + v_d^2 + \frac{9}{4}v_e^2\right) & \frac{g_1 g_2}{4}\left(-\frac{1}{2}v_u^2 - v_d^2 + \frac{3}{2}v_e^2\right) & \frac{g_1 g_4}{8\sqrt{6}}\left(5v^2 - v_u^2 + 9v_e^2\right) \\ \frac{g_1 g_2}{4}\left(-\frac{1}{2}v_u^2 - v_d^2 + \frac{3}{2}v_e^2\right) & \frac{g_2^2}{4}\left(3v_u^2 + v_d^2 + v_e^2\right) & \frac{3g_2 g_4}{4\sqrt{6}}\left(v_u^2 + v_e^2\right) \\ \frac{g_1 g_4}{8\sqrt{6}}\left(5v^2 - v_u^2 + 9v_e^2\right) & \frac{3g_2 g_4}{4\sqrt{6}}\left(v_u^2 + v_e^2\right) & \frac{g_4^2}{8}\left(v^2 + v_u^2 + 3v_e^2\right) \end{pmatrix} \tag{8}$$

smaller. v_d tends to be the largest among v_d, v_u and v_e, thus playing approximately the role of the Standard Model vev.

The Higgs vevs induce the following vector boson masses:

- Axigluon mass: $\frac{1}{2}m_N^2 = \frac{1}{2}(g_3^2 + g_4^2)(v^2 + v_u^2)$
  This mass is always large because it involves the large vev v and both strong couplings g_3 and g_4.
- Leptoquark mass: $\frac{1}{2}m_V^2 = \frac{1}{4}g_4^2(v^2 + v_u^2 + v_e^2)$
  This mass is in general smaller than the axigluon mass because a term $\sim g_3^2$ is missing (assuming $v_e \ll v$). However, it competes with the mass of the neutral boson related to the ρ^{15} generator, as discussed below.
- Mass matrix of the neutral vector bosons C_μ, W_μ^3 and S_μ: (see (8) on top of the page). Note that this matrix has one vanishing and two nonvanishing eigenvalues. The corresponding eigenstates will be called A_μ, Z_μ and T_μ. By calculating the characteristic polynomial, one sees that the mass of the state T_μ is governed by the vev v whereas m_Z is independent of v and thus smaller. The rotation matrix which diagonalizes $\frac{1}{2}M^2$ will be called r, i.e. $r^T \frac{1}{2}M^2 r = \text{diag}(0, \frac{1}{2}m_Z^2, \frac{1}{2}m_T^2)$.
- $W^\pm$ mass: $\frac{1}{2}m_W^2 = \frac{1}{4}g_2^2(3v_u^2 + v_e^2 + v_d^2)$
  This is like in the Standard Model with $v_{SM} \approx 175$ GeV replaced by $3v_u^2 + v_e^2 + v_d^2$. When diagonalizing the mass matrix of the neutrals, it will be possible to maintain the relation $m_W = c_W m_Z$ where c_W is the cosine of the ordinary Weinberg angle.

6 Electroweak sector

The mass matrix of the neutral gauge bosons has to fulfill two requirements. First of all, the Z-mass must come out as $\frac{1}{2}m_Z^2 = \frac{e^2}{4s_W^2 c_W^2}(3v_u^2 + v_e^2 + v_d^2)$ to fulfill $m_W = c_W m_Z$. Secondly, the couplings of photon and Z, which are linear combinations of C_μ, W_μ^3 and S_μ, must be as in the standard model. For example, the coupling of the Z to left-handed fermions f_L must be $\frac{e}{s_W c_W}(T_f^3 - s_W^2 Q_f)$. In these relations, the three quantities e, $s_W = \sqrt{1 - c_W^2}$ and Q still have to be defined in the framework of our model. s_W is simply $s_W = \frac{e}{g_2}$, as in the standard model. e and Q are properties of the massless photon state A_μ and given by

$$\frac{e^2}{g_1^2} + \frac{e^2}{g_2^2} + \frac{25}{6}\frac{e^2}{g_4^2} = 1 \tag{9}$$

and

$$Q = X - \frac{5}{\sqrt{6}}T_{15} + T_3 \tag{10}$$

because the photon state is given by

$$\frac{A_\mu}{e} = \frac{C_\mu}{g_1} + \frac{W_\mu^3}{g_2} - \frac{5}{\sqrt{6}}\frac{S_\mu}{g_4} \tag{11}$$

Note that $T_{15} = \frac{\rho_{15}}{2}$ and $T_3 = \frac{\tau_3}{2}$. Furthermore, the coupling g_Y corresponding to the Standard Model weak hypercharge is given by

$$g_Y^{-2} = g_1^{-2} + \frac{25}{6}g_4^{-2} . \tag{12}$$

Given $e = 0.303$ and $g_2 = 0.636$, (9) can be considered as a relation between g_1 and g_4. One may introduce an angle ϕ by the relations

$$c_\phi = \frac{e}{g_1 c_W} \qquad s_\phi = \frac{5e}{\sqrt{6}g_4 c_W} \tag{13}$$

with $c_\phi^2 + s_\phi^2 = 1$. This angle will simplify the notation in the following.

The requirement that the couplings of photon and Z must be as in the standard model, completely fixes the rotation matrix r which must be used to diagonalize $\frac{1}{2}M^2$, (8). It can be shown that all photon and Z couplings come out in agreement with the Standard Model if and only if one has

$$r = \begin{pmatrix} c_W c_\phi & -s_W c_\phi & s_\phi \\ s_W & c_W & 0 \\ -c_W s_\phi & s_W s_\phi & c_\phi \end{pmatrix} \tag{14}$$

In general this matrix r will *not* diagonalize $\frac{1}{2}M^2$, and thus in general the Standard Model couplings cannot be reproduced. However, there is one simple condition under which r, (14), completely diagonalizes $\frac{1}{2}M^2$ and at the same time gives the correct Z mass, namely

$$v_u^2 + v_e^2 = \frac{2}{5}s_\phi^2(3v_u^2 + v_e^2 + v_d^2) = \frac{4}{5e^2}s_\phi^2 s_W^2 c_W^2 m_Z^2 \tag{15}$$

This condition can be fulfilled for various values of the vevs. A typical solution is v being larger than v_d and this in turn is larger than v_u and v_l. However, it should be noted that one must not take $v_u = v_e = 0$, because otherwise $g_4 \to \infty$ according to (15). Within this solution, the leptoquark mass is always of the same order as the mass of the T-particle (to within ± 50 GeV), whereas the axigluon masses can be made higher than 1 TeV (if desired). Note that there are Tevatron limits on the mass of neutral vector bosons, of about 550 GeV [17]. This fact can be used to argue that the leptoquark in our model should have mass $m_V \gtrsim 500$ GeV.

Due to the mixing of photon, Z and T, there is another constraint on the coupling g_4 (weaker than $g_4 \geq g_s$), arising from the limit $s_\phi \leq 1$ in the electroweak sector. It

Table 2. Anomalies of the model. The generators of $SU(4)$, $SU(3)$, $SU(2)_L$ and $U(1)_X$ are denoted by ρ, λ, τ and X, respectively. Note that the anomaly for τ^3 vanishes as a consequence of a general $SU(2)$ property, and the anomalies for $X^2\tau$, $X^2\lambda$ and $X^2\rho$ are zero due to the tracelessness of the $SU(2)$, $SU(3)$ and $SU(4)$ generators

	X^3	$X\tau^2$	ρ^3	λ^3	$X\rho^2$	$X\lambda^2$
q_L	$\frac{1}{36}$	$\frac{1}{4}$	0	2	0	$\frac{1}{6}$
l_L	$-\frac{1}{4}$	$-\frac{1}{4}$	0	0	0	0
p_R	$-\frac{1}{16}$	0	-1	0	$-\frac{1}{8}$	0
d_R	$\frac{1}{9}$	0	0	-1	0	$\frac{1}{6}$
	$-\frac{25}{144}$	0	-1	1	$-\frac{1}{8}$	$\frac{1}{3}$

Table 3. Anomaly contributions of the new fermions. The same notation as in Table 2 is used

	X^3	$X\tau^2$	ρ^3	λ^3	$X\rho^2$	$X\lambda^2$
F_L	$\frac{1}{16}$	0	1	0	$\frac{1}{8}$	0
G_R	$-\frac{8}{9}$	0	0	-1	0	$-\frac{1}{3}$
K_R	1	0	0	0	0	0
	$\frac{25}{144}$	0	1	-1	$\frac{1}{8}$	$-\frac{1}{3}$

is given by $g_4 \geq \frac{5e}{\sqrt{6}c_W} \approx 0.70$. Conversely, the condition $g_4 \geq g_s$ translates into a constraint for s_ϕ, namely $s_\phi^2 \lesssim 0.5$.

It might be interesting to examine the Higgs content of the various vector bosons, for simplicity in the limit $v \gg v_d \gg v_{u,l}$. In that limit 3 of the 4 real components of H_d are eaten up by W and Z leaving the Standard Model Higgs field as a real particle. The longitudinal components of the axigluons, leptoquarks and neutral (8+6+1=15 real degrees of freedom) are given by 15 components of the Higgs multiplet $H(\bar{4},3,1)$, namely Im $H_{\bar{\alpha}\beta}$ ($\alpha,\beta = 1,2,3$), $H_{\bar{1}\beta}$ ($\beta = 1,2,3$) and Im $H_{\bar{\beta}\beta}$, respectively. The 9 remaining real parts of $H_{\bar{\alpha}\beta}$ and $H_{\bar{\beta}\beta}$ will be real Higgs particles.

7 Checking the vector boson self interactions

Besides its interactions with leptons and quarks, the leptoquark interacts with other Standard Model particles, namely with the photon, the gluon and the Z. This can be seen by working out the terms $-\frac{1}{4}R_{\mu\nu}^a R^{\mu\nu a} - \frac{1}{4}L_{\mu\nu}^a L^{\mu\nu a}$ in the Lagrangian. Not surprisingly, one finds that the coupling strength to the photon is $\frac{5}{3}e$ and to the gluons is given by g_s. The coupling to gluons is being used in the Tevatron searches for leptoquarks via the process $q\bar{q} \rightarrow g^* \rightarrow V\bar{V}$. Note that the Yang–Mills terms induce several other interesting vector boson self couplings which will not be discussed here.

8 Anomaly cancellation and unification

It is well known that in the Standard Model all γ_5-anomalies cancel. In the present model this does not happen, unless additional exotic fermion multiplets are introduced. As is shown below, this works similarly as in the chiral–color models based on $SU(3)_L \times SU(3)_R$ [18].

The list of anomaly coefficients of the standard fermions is given in Table 2. They are summed up in the last line of this table. Additional fermion multiplets have to be chosen in such a way that their contributions exactly cancel the numbers in the last line of Table 2. We

have scanned through all possible fermion representations of $SU(4) \times SU(3) \times SU(2)_L \times U(1)_X$ and have obtained a very simple solution to this problem in the form of three additional fermion multiplets, namely a left-handed $SU(4)$ quartet $F_L(4,1,1,\frac{1}{4})$, a $SU(3)$ triplet $G_R(1,3,1,\frac{2}{3})$ and a singlet state $K_R(1,1,1,-1)$. Note that F_L and G_R have similar but not identical quantum numbers to the standard fermions p_R and d_R, respectively. In Table 3 the contributions of these new fermions to the various anomaly coefficients are given. The last line of the table sums up these contributions. By comparing the last lines of Tables 2 and 3 it can be seen that all the anomalies completely cancel. It should be noted that the additional fermions have been chosen to be singlets under $SU(2)_L$. This makes sure that the cancellation of the $SU(2)_L$ anomalies (second row of Table 2) is not disturbed. Note further, that a family repetition structure of the standard fermions and of the new fermions is understood in all the considerations.

The next step is to generate mass terms for the new fermions. These masses have to be large enough to avoid conflict with existing bounds on heavy fermions. Since F_L, G_R and K_R are $SU(2)_L$ singlets, the particularly strong constraints on $SU(2)_L$ doublets do not apply [19]. Therefore, masses of the new fermions larger than 500 GeV are certainly compatible with all present limits [19]. In the following we want to describe how masses in the range between 500 and 1000 GeV can be obtained. The singlet property of F_L and G_R under $SU(2)_L$ has the convenient consequence, that it allows to write down a Yukawa coupling term of the form $\bar{G}_R F_L H(\bar{4},3,1,\frac{5}{12})$, where $H(\bar{4},3,1,\frac{5}{12})$ is the Higgs multiplet with vev $v\delta_{\bar{\alpha}\beta}$ used earlier to break $SU(4) \times SU(3)$ to $SU(3)_c$. This Yukawa term will give a mass to G_R and the first three components of F_L which is of the order v, i.e. the scale of the leptoquark mass. To obtain a mass term for K_R and the fourth component of F_L, an additional Higgs field $H_K(\bar{4},1,1,-\frac{5}{4})$ with vev $v_K\delta_{\bar{\alpha}4}$ has to be introduced. v_K must be of the same order of magnitude as v if all the new fermions are to be (at least) as heavy as the leptoquarks. Thus, the Yukawa interactions of the new fermions are given by

$$L_{Yuk,new} = hH\bar{G}_R F_L + h_K H_K \bar{K}_R F_L + c.c. \qquad (16)$$

where h and h_K are the corresponding Yukawa coupling parameters, which should be chosen of the order $O(1)$. The reader should remember that v (and v_K) were assumed to be larger than the vevs v_u, v_d and v_e which gave masses

to the ordinary fermions. Therefore with (16), the masses of the new fermions are much larger than those of the standard fermions.

What about mixing terms? Having fixed the set of Higgs multiplets, one should in principle write down all possible Yukawa interactions which are compatible with the symmetries of the model. In fact there exist only two Yukawa interactions in addition to those already introduced in (7) and (16), namely $h_1 H_d \bar{G}_R q_L$ and $h_2 H_d \bar{l}_L K_R$, where h_1 and h_2 denote the coupling strengths. If one adds them to (7) and (16) and inserts the vacuum expectation values, one obtains the complete set of fermion mass terms

$$L_m = h v \bar{U}_R U_L + h_K v_K \bar{E}_R E_L + h_e v_e \bar{e}_L e_R + h_u v_u \bar{u}_L u_R$$
$$+ h_d v_d \bar{d}_L d_R + h_1 v_d \bar{U}_R u_L + h_2 v_d \bar{e}_L E_R + c.c. \quad (17)$$

In this equation we have introduced the notation $F_L = (U_L, E_L)$, $G_R = U_R$ and $K_R = E_R$. One concludes from (17) that the mass eigenstates are d, e', E', u' and U', where e' and E' are linear combinations of $e = e_L + e_R$ and $E = E_L + E_R$, and u' and U' are linear combinations of $u = u_L + u_R$ and $U = U_L + U_R$. However, working in the limit that v and v_K are larger than v_u, v_d and v_e, the mixing angle is small, so that roughly $U' \sim U$, $E' \sim E$, $u' \sim u$ and $e' \sim e$. Therefore, the inclusion of the new fermions influences the sector of the standard fermions only marginally.

Another point to discuss is that, introducing a new Higgs field H_K with a vev that breaks $SU(4)$, one has to make sure that the previously derived properties of the symmetry breaking in the gauge sector are not spoiled. We have analyzed this problem and found that the only modifications induced by H_K concern the mass formulae for the leptoquark and for the heavy neutral state T. All other features of the dynamical symmetry breaking, like the relations for the photon and the mass ratio $\frac{m_W}{m_Z}$, remain intact. The modified leptoquark mass is given by

$$\frac{1}{2} m_V^2 = \frac{1}{4} g_4^2 (v^2 + v_K^2 + v_u^2 + v_e^2) \quad (18)$$

whereas the axigluon mass is not modified. Thus the additional Higgs field H_K tends to increase the leptoquark mass, although mass values below 1 TeV are still consistent with all requirements. Similarly, H_K induces an increase in the value of $\frac{1}{2} m_T^2$ by an amount $\frac{3}{8} g_4^2 v_K^2$.

The actual mass values of the new fermions can be chosen rather freely by the choice of the Yukawa coupling. However, large Yukawa cuplings of order $O(1)$ are more appropriate than small ones, because the new fermion masses are hv and $h_K v_K$, respectively and should be roughly of the order of the leptoquark mass $m_V \approx g_4 \sqrt{\frac{1}{2}(v^2 + v_K^2)}$ where $g_4 \sim 1$ and $v \sim v_K$.

At this point it should perhaps be stressed that, even including the new fermion multiplets, our model is not as complicated as it may appear. It is essentially the Standard Model with the main modification that the right-handed up-type quarks and leptons form a $SU(4)$ representation. All other features of the model then follow from consistency requirements, like gauge principle, universality, anomaly cancellation etc.

Cancelling the anomalies by new fermions is a rather ad–hoc procedure (although quite common in model building). It would be more interesting if the anomalies could be cancelled by embedding the $SU(4) \times SU(3) \times SU(2)_L \times U(1)_X$ in an anomaly free grand unified theory. Unfortunately, the group is so large, that only GUT groups of rank ≥ 7 like E_7, E_8 or $SO(18)$ with inconveniently large fermion representations are possible. Therefore, unification is not a straightforward option in our model. In any case, the new particles – leptoquarks, axigluons etc. – modify the running of coupling constants, so that the GUT scenario is strongly modified by the additional $SU(4)$ symmetry.

9 Summary

We believe that our model has general implications on gauge models with vector leptoquarks of mass ≤ 1 TeV. Therefore as a summary we want to stress its general features, which are independent of the chosen symmetry breaking mechanism. One of them is the appearance of an SU(4) quartet (u_R, e_R) with couplings to leptoquarks. Any other combination, involving e.g. d–quarks, would be in conflict with low energy constraints.

Among the SU(4) gauge bosons there are, besides leptoquarks, necessarily neutral as well as gluon–like fields whose masses are of the order of the leptoquark mass. This is enforced in order to close the color algebra. The gluonic type gauge bosons will mix with the ordinary gluons, and relations of the form $g_s = g_{LQ} \cos \theta$ will appear through this mixing forcing the leptoquark coupling to be a strong coupling, $g_{LQ} \geq g_s$.

Within the proposed symmetry breaking scheme, it has turned out that there is one Higgs field which strongly resembles the Standard Model Higgs particle. Further, there is another one which is mainly responsible for the breaking of the SU(4) symmetry.

In the present model, the right-handed up-type quarks are part of the SU(4) quartets whereas the right-handed down-type quarks are SU(4) singlets. It is thus apparent that the custodial symmetry between u_R and d_R which is respected by the Standard Model (neglecting quark masses) is violated. Correspondingly, one expects "large" loop corrections to the ρ-parameter [20] (or $\epsilon_{1,2,3}$ [21]) other than $\sim m_t^2 - m_b^2$. In fact, there are additional self-energy diagrams of W and Z with either leptoquarks $V^\pm$, the neutral Z' (T_μ) as well as many of the Higgs components discussed above. In addition, there are the new fermions needed for anomaly cancellation. Of course one can always argue that the contribution of these particles to $\epsilon_{1,2,3}$ is sufficiently small if their masses are larger than, say, 500 GeV [21].

Appendix

Finally we want to discuss what happens if the assumption of vanishing fermion number is given up. Low en-

Table 4. Quantum number assignments of the $F = -2$ lepto-quark model

	$SU(5)$	$SU(3)$	$SU(2)_q$	$U(1)_X$
q_L	1	3	2	$\frac{1}{6}$
p_L	5	1	1	$\frac{1}{5}$
e_R	1	1	1	-1
d_R	1	3	1	$-\frac{1}{3}$

ergy constraints, universality and gauge principle then allow for a leptoquark interaction of the form $\bar{u}_R^c \gamma^\mu l_L U_\mu$, where U_μ is an $SU(2)_L$ doublet of vector leptoquarks. Using the same philosophy as before, one may now construct a gauge theory based on $SU(5) \times SU(3) \times SU(2)_q \times U(1)_X$ where the left-handed quarks q_L transform as a doublet under $SU(2)_q$ whereas the left-handed leptons are part of a $SU(5)$ quintet[2], $p_L \equiv (u_R^{1c}, u_R^{2c}, u_R^{3c}, \nu_L, e_L)$. Thus the $SU(5)$ contains a subgroup $SU(3)' \times SU(2)_l$ where $SU(3)' \times SU(3) \to SU(3)_c$ as before, giving rise to 8 massive axigluons, and $SU(2)_l \times SU(2)_q \to SU(2)_L$, introducing 3 additional massive states, the "axi-$W^\pm/Z$". There is also a neutral gauge boson S which mixes with the photon. The remaining 12 $SU(5)$ gauge bosons constitute the leptoquark U_μ.

To give some more details of the model, the quantum number assignments of the standard fermions are shown in Table 4. The $U(1)$ charge of the quintet can be fixed to be 1/5 by the requirement that the photon coupling is vectorlike. This as well as many other features of the model work out in the same way as the $SU(4)$ model presented in the main text. For example, one now has

$$Q = X - \frac{7}{\sqrt{15}} T_{24} + T_3^q + T_3^l \tag{19}$$

where $T_{24} = \frac{1}{\sqrt{15}} \mathrm{diag}(-1, -1, -1, \frac{3}{2}, \frac{3}{2})$ and T_3^l and T_3^q are the diagonal generators of $SU(5)$ and $SU(2)_q$, respectively. Note that the $SU(5)$ gauge bosons R_μ are decomposed into $SU(3)'$ gauge bosons R'_μ, $SU(2)_l$ gauge bosons W_μ^l, the leptoquarks U_μ and a singlet S_μ, which is related to the T_{24} generator. More precisely, one has

$$R_\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} R'_\mu - \frac{2}{\sqrt{30}} S_\mu \times 1_3 & U_\mu \\ U_\mu^+ & W_\mu^l + \frac{3}{\sqrt{30}} S_\mu \times 1_2 \end{pmatrix} . \tag{20}$$

The group $SU(5) \times SU(3) \times SU(2)_q$ can be broken to $SU(3)_c \times SU(2)_L$ by two Higgs multiplets, $H_3(\bar{5}, 3, 1, -\frac{7}{15})$ with vev $v_3 \delta_{\bar{\alpha}\beta}$ and $H_2(\bar{5}, 1, 2, \frac{7}{10})$ with vev $v_2 \delta_{\bar{\alpha}i}$, where α, β and i denote $SU(5)$, $SU(3)$ and $SU(2)_q$ indices, respectively. The following vector boson masses are then obtained:

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[2] Note that the quintet is reminiscent of the quintet in "flipped $SU(5)$" [22]. However, in contrast to flipped $SU(5)$, here one has no 10–representation containing d–quarks because this would induce proton decay

- Axigluon mass: $\frac{1}{2} m_N^2 = \frac{1}{2}(g_3^2 + g_5^2) v_3^2$
- Axi-$W^\pm/Z$ mass: $\frac{1}{2} m_M^2 = \frac{1}{2}(g_{2q}^2 + g_5^2) v_2^2$
- Leptoquark mass: $\frac{1}{2} m_U^2 = \frac{1}{4} g_5^2 (v_2^2 + v_3^2)$
- Neutral vector boson mass: $\frac{1}{2} m_S^2 = \frac{1}{2} g_5^2 (\frac{3}{5} v_2^2 + \frac{2}{5} v_3^2)$

In these expressions, g_5, g_3 and g_{2q} denote the couplings of $SU(5)$, $SU(3)$ and $SU(2)_q$, respectively. Note that one has

$$g_s^{-2} = g_3^{-2} + g_5^{-2} \tag{21}$$

$$g_2^{-2} = g_{2q}^{-2} + g_5^{-2} \tag{22}$$

and

$$g_Y^{-2} = g_X^{-2} + \frac{49}{15} g_5^{-2} \tag{23}$$

where g_s, g_2 and g_Y are the couplings of the Standard Model gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$. Equations (21), (22) and (23) are in analogy to (6) and (12) for the $SU(4)$-type model. They imply that in general $g_{2q} \ll g_{3,5}$. This suggests that the axi-$W^\pm/Z$ mass might be somewhat smaller than the axigluon mass, although this is not compelling because the ratio of these masses depends also on the relative magnitude of v_2 and v_3.

As in the $SU(4)$ model, the fermion masses m_e, m_u and m_d can be obtained from three Higgs fields, $H_e(5, 1, 1, \frac{6}{5})$, $H_u(5, 3, 2, \frac{11}{30})$ and $H_d(1, 1, 2, \frac{1}{2})$ with vevs $v_e \delta_{\alpha 5}$, $v_u \delta_{\alpha\beta} \delta_{i1}$ and $v_d \delta_{i2}$. These expectation values in principle contribute to the vector boson masses as well, so that a complicated mixing matrix as in (8) arises. However, just as in the $SU(4)$ model, it turns out that $v_{e,u,d} \ll v_{2,3}$, so that the mass formulas given before (21) are still approximately valid (up to terms of order $v_{e,u,d}^2$). As in the $SU(4)$ model, the vevs $v_{e,u,d}$ determine not only the fermion masses but also the mass of the W- and Z-boson, where by use of (19) one is lead to the correct value of the Weinberg angle.

References

1. H1 Collaboration, hep-ex/9702012; ZEUS Collaboration, hep-ex/9702015
2. G. Altarelli, J. Ellis, G.F. Guidice, S. Lola, M.L. Mangano, hep-ph/9703276
3. J. Kalinowski, R. Rückl, H. Spiesberger, P.M. Zerwas, hep-ph/9703288
4. V. Barger, K. Cheung, K. Hagiwara, D. Zeppenfeld, hep-ph/9703311; M.C. Gonzales, S.F. Novaes, hep-ph/9703346; N. di Bartolomeo, M. Fabbrichesi, hep-ph/9703375; D. Choudhury, S. Raychaudhuri, hep-ph/9702392; J. Blümlein, hep-ph/9703287; H. Dreiner, P. Morawitz, hep-ph/9703279; J.L. Hewett, T.G. Rizzo, hep-ph/9703337; E. Perez, Y. Sirois, H. Dreiner, hep-ph/9703444; T. Kon, T. Kobayashi, hep-ph/9704221; R. Barbieri, Z. Berezhiani, A. Strumia, hep-ph/9704275; J. Ellis, S. Lola, K. Sridhar, hep-ph/9705416
5. F. Abe et al. (CDF Collaboration), fermilab-pub-93/070-E
6. J.C. Pati, A. Salam, Phys. Rev. **D10**, 275 (1974)

7. H. Fritzsch, P. Minkowski, Ann. Phys. **93**, 193 (1975)
8. S. Davidson, D. Bailey, B.A. Campbell, Z. Phys. **C61**, 613 (1994)
9. O. Shanker, Nucl. Phys. **B206**, 253 (1982)
10. M. Leurer, Phys. Rev. **D49**, 333, (1994) **D50**, 536 (1994); A. Deandrea, hep-ph/9705435; L. Giusti, A. Strumia, hep-ph/9706298
11. K.S. Babu, C. Kolda, J. March-Russell, F. Wilczek, hep-ph/9703299
12. W. Buchmüller, R. Rückl, D. Wyler, Phys. Lett. **B191**, 442 (1987)
13. J.L. Hewett, in *Proc. of the 1990 Summer Study on High Energy Physics*, Snowmass; T. Kon, T. Kobayashi, Phys. Lett. **B270**, 81 (1991); J. Butterworth, H. Dreiner, Nucl. Phys. **B397**, 3 (1993)
14. P.H. Frampton, S.L. Glashow, Phys. Lett. **B190**, 157 (1987); P.H. Frampton, S.L. Glashow, Phys. Rev. Lett. **58**, 2168 (1987); F. Cuypers, Acta Phys. Polon. **B22**, 137 (1991); S. Rajpoot, Phys. Rev. **D60**, 2003 (1988)
15. F. Abe et al. (CDF Collaboration), Phys. Rev. Lett. **71**, 2542 (1993)
16. J. Bagger, C. Schmidt, S. King, Phys. Rev. **D37**, 1188 (1987)
17. K. Maeshima (CDF Collaboration), in *Proc. of the 1996 International Conference on High Energy Physics*, Warsaw, 1996
18. S. Rajpoot, Phys. Rev. **D38**, 417 (1988); R. Foot, H. Lew, R.R. Volkas, G.C. Joshi, Phys. Lett. **B217**, 301 (1988); C.Q. Geng, Phys. Rev. **D39**, 2402 (1989)
19. Review of Particle Properties, 1996, by Particle Data Group
20. M. Veltman, Nucl. Phys. **B123**, 89 (1977)
21. G. Altarelli, R. Barbieri, S. Jadach, Nucl. Phys. **B369**, 3 (1992); G. Altarelli, R. Barbieri, F. Caravaglios, Nucl. Phys. **B405**, 3 (1993)
22. S. Barr, Phys. Lett. **B112**, 219, (1982) Phys. Rev. **D40**, 2457 (1989); I. Antoniadis, J. Ellis, J. Hagelin, D.V. Nanopoulos, Phys. Lett. **B194**, 231 (1987)